№1 3 стандартные, 2 нестандартные

успех: 2 стандартные

$$\mathbb{P}\{\xi = 2\} = \frac{3}{5}\cdot\frac{2}{4} = \frac{3}{10}$$

$$\mathbb{P}\{\xi = 3\} = \frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3} + \frac{2}{5}\cdot\frac{3}{4}\cdot\frac{2}{3} = \frac{4}{10}$$

$$\mathbb{P}\{\xi = 4\} = \frac{2}{5}\cdot\frac{1}{4} + \frac{2}{5}\cdot\frac{3}{4}\cdot\frac{1}{3} + \frac{3}{5}\cdot\frac{2}{4}\cdot\frac{1}{3} = \frac{3}{10}$$

ξ:

2	3	4
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{3}{10}$

№2

$\eta \backslash \xi$	1	2	3	4	5	6
1	$\frac{1}{36}$	0	0	0	0	0
2	$\frac{1}{36}$	$\frac{2}{36}$	0	0	0	0
3	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{3}{36}$	0	0	0
4	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{4}{36}$	0	0
5	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{5}{36}$	0
6	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{6}{36}$

ξ:

1	2	3	4	5	6
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

η:

1	2	3	4	5	6
$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$$\mathrm{cov} = E\xi\eta - E\xi E\eta = \frac{35}{24}$$

№3

$$p_\xi(x) = \begin{cases} c(1-|x|), & x \in [-1, 1] \\ 0, & \text{иначе} \end{cases}$$

$$P_\xi(x) = \begin{cases} c(1-x), & x \in [0,1] \\ c(1+x), & x \in [-1,0] \\ 0, & \text{иначе} \end{cases}$$

$$\int_0^1 c(1-x)dx + \int_{-1}^0 c(1+x)\,dx = \int_{-1}^0 c(1+x)\,d(x+1) -$$

$$- \int_0^1 c(1-x)\,d(-x) = \frac{c(1+x)^2}{2}\Bigg|_{-1}^0 - \frac{c(1-x)^2}{2}\Bigg|_0^1 =$$

$$= \frac{c}{2} + \frac{c}{2} = c = 1 \Rightarrow c = 1$$

$F_\xi(x)$ – ?

1) $x \in [-1, 0]$

$$F_\xi(x) = \int_{-1}^x (1+t)\,dt = \frac{(1+t)^2}{2}\Bigg|_{-1}^x =$$

$$= \frac{(1+x)^2}{2}$$

2) $x \in (0, 1]$

$$F_\xi(x) = \int_{-1}^0 (1+t)dt + \int_0^x (1-t)\,dt =$$

$$= \frac{(1+t)^2}{2}\Big|_{-1}^{0} - \frac{(1-t)^2}{2}\Big|_{0}^{x} = \frac{1}{2} - \frac{(1-x)^2}{2} + \frac{1}{2} =$$

$$= 1 - \frac{(1-x)^2}{2}$$

$$F_\xi(x) = \begin{cases} 0, & x < -1 \\ \frac{(1+x)^2}{2}, & x \in (-1, 0] \\ 1 - \frac{(1-x)^2}{2}, & x \in (0, 1] \\ 1, & x > 1 \end{cases}$$

$$\mathbb{P}\{-0{,}25 \leq \xi \leq 0{,}5\} = F_\xi(0{,}5) - F_\xi(-0{,}25) =$$

$$= 1 - \frac{(1-0{,}5)^2}{2} - \frac{(1-0{,}25)^2}{2} = \frac{19}{32}$$

$$y = |\xi| - 1, \quad t \in (-1, 0)$$

$$F_y(t) = \mathbb{P}\{|\xi| - 1 < t\} = \mathbb{P}\{|\xi| < t+1\} =$$

$$= \mathbb{P}\{-1-t < \xi < t+1\} =$$

$$= F_\xi(t+1) - F_\xi(-1-t) =$$

$$p_y(t) = p_\xi(t+1) + p_\xi(-1-t) = \begin{cases} 1 - |t+1| + 1 - |-1-t), & t \in (-1, 0] \\ 0, \ \ldots \end{cases} =$$

0, иначе

$$= \begin{cases} -2x, & x \in [-1,0] \\ 0, & \text{иначе} \end{cases}$$

$$E\eta = \int_{-1}^{0} -2x^2\,dx = -\frac{2x^3}{3}\Big|_{-1}^{0} = -\frac{2}{3}$$

$$E\eta^2 = \int_{-1}^{0} -2x^3\,dx = -\frac{x^4}{2}\Big|_{-1}^{0} = \frac{1}{2}$$

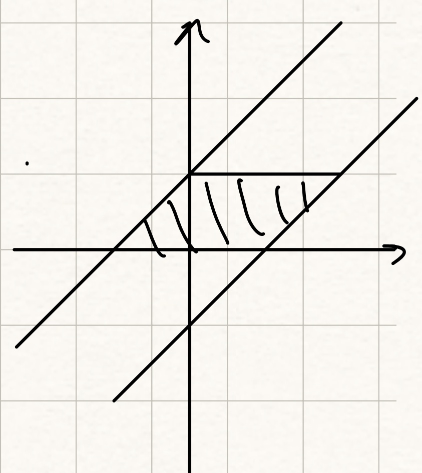
$$D\eta = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$$

№4. $\xi \sim U[0,1]$, $\eta \sim U[-1,1]$

$$\zeta = \xi - \eta$$

$$p_\zeta(t) = \int_{-\infty}^{+\infty} p_\xi(u)\,p_\eta(u-t)\,du = I$$

$$\begin{cases} 0 \le u \le 1 \\ -1 \le u - t \le 1 \end{cases} \Rightarrow \begin{cases} 0 \le u \le 1 \\ t-1 \le u \le t+1 \end{cases}$$



1) $t \in [-1, 0]$

$$I = \int^{t+1} \frac{1}{2}\,du = \frac{t+1}{2}$$

2) $t \in [0, 1]$ $\quad I = \int\limits_0^1 \frac{1}{2}\, du = \frac{1}{2}$

3) $t \in [1, 2]$ $\quad I = \int\limits_{t-1}^{1} \frac{1}{2}\, du = 1 - \frac{t}{2}$

$$P_y(t) = \begin{cases} \frac{t+1}{2}, & t \in [-1, 0) \\ \frac{1}{2}, & t \in [0, 1] \\ 1 - \frac{t}{2}, & t \in [1, 2] \\ 0, & \text{иначе} \end{cases}$$

$$Ey = \int\limits_{-1}^{0} \frac{t^2+t}{2}\, dt + \int\limits_0^1 \frac{t}{2}\, dt + \int\limits_1^2 t - \frac{t^2}{2}\, dt =$$

$$= -\frac{1}{12} + \frac{1}{4} + \frac{1}{3} = \frac{1}{2}$$

$$Ey^2 = \int\limits_{-1}^{0} \frac{t^3+t^2}{2}\, dt + \int\limits_0^1 \frac{t^2}{2}\, dt + \int\limits_1^2 t^2 - \frac{t^3}{2}\, dt =$$

$$= \frac{1}{24} + \frac{1}{6} + \frac{11}{24} = \frac{2}{3}$$

$$Dy = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

№5 $\quad p(x_1, x_2) = \frac{1}{2\pi} e^{-\frac{x_1^2 + x_2^2}{2}}$

$$\begin{cases} \eta_1 = \xi_1 - \xi_2 \\ \eta_2 = \xi_1 + \xi_2 \end{cases} \Rightarrow \begin{cases} \xi_1 = \frac{\eta_1 + \eta_2}{2} \\ \xi_2 = \frac{\eta_2 - \eta_1}{2} \end{cases}$$

$$J = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$p_{y_1, y_2}(y_1, y_2) = \frac{1}{2} p_{\xi_1, \xi_2}\left(\frac{y_1 + y_2}{2}, \frac{y_2 - y_1}{2}\right) =$$

$$= \frac{1}{4\pi} \exp\left(\frac{-\left(\frac{y_1 + y_2}{2}\right)^2 - \left(\frac{y_2 - y_1}{2}\right)^2}{2}\right) =$$

$$= \frac{1}{4\pi} \exp\left(-\frac{(y_1 + y_2)^2 + (y_2 - y_1)^2}{8}\right) =$$

$$= \frac{1}{4\pi} \exp\left(-\frac{y_2^2 + y_1^2}{4}\right)$$

$$p_{\eta_1} = \int_{-\infty}^{+\infty} \frac{1}{4\pi} e^{-\frac{y_2^2 + y_1^2}{4}} dy_2 = \frac{1}{4\pi} e^{-\frac{y_1^2}{4}} \int_{-\infty}^{+\infty} e^{-\frac{y_2^2}{4}} dy_2$$

$$= \frac{1}{2\sqrt{\pi}} e^{-\frac{y_1^2}{4}} \int_{-\infty}^{+\infty} \frac{1}{2\sqrt{\pi}} e^{-\frac{y_2^2}{4}} dy_2 = \frac{1}{2\sqrt{\pi}} e^{-\frac{y_1^2}{4}} -$$

– норм.

расп.-е

$E y_1 = 0, \ D y_1 = 2$

y_1, y_2 - незав.-е $\Rightarrow \operatorname{cov}(y_1, y_2) = 0$